DO NOW

pg 327; 10

$$\int_1^3 (3x^2+5x-4)\,dx$$

$$\left[x^3+\tfrac{5}{2}x^2-4x\right]_1^3$$

$$\left(27+\tfrac{5}{2}\cdot 9-12\right)-\left(1+\tfrac{5}{2}-4\right)$$

$$27+\tfrac{45}{2}-12-1-\tfrac{5}{2}+4$$

$$18+20$$

$$\boxed{38}$$

5.4 The Fundamental Theorem of Calculus - Day 2

Using the calculator and definite integrals:

1. Algebraically:

$\boxed{\text{MATH}}$ $\boxed{9}$ fnInt (f(x), x, lower, upper)

2. Graphically:

Graph ; Calc ; $\boxed{7}$ $\int f(x)dx$

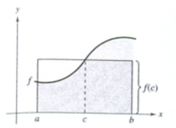
enter bounds

The Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that:

$$\int_a^b f(x)\,dx = \underset{\substack{\uparrow \\ \text{length} \\ \text{(height)}}}{f(c)}\;\underset{\substack{\uparrow \\ \text{width}}}{(b-a)}$$

Area on curve = $\overbrace{\text{length}\cdot\text{width}}$



Mean value rectangle:
$f(c)(b-a) = \int_a^b f(x)\,dx$

***This is an existence theorem...

doesn't find "c" ... it just guarantees that "c" exists.

Average Value Function

If f is integrable on the closed interval $[a, b]$, then:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

*Use to find c

Example: Find the average value of $f(x) = 3x^2 - 2x$ on $[1, 4]$.

$$f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f(c)=\frac{1}{4-1}\int_1^4 (3x^2-2x)\,dx$$

$$f(c)=\frac{1}{3}\left[x^3-x^2\right]_1^4$$

$$f(c)=\frac{1}{3}\left[(64-16)-(1-1)\right]$$

$$f(c)=\frac{1}{3}(48)$$

$$f(c)=16 \longleftrightarrow \text{average height}$$

Find c for the above average value of f.

$$3x^2-2x=16$$

$$3x^2-2x-16=0$$

$$(3x-8)(x+2)=0$$

$$x=\frac{8}{3} \qquad \cancel{x=-2}$$

interval

$$\boxed{\therefore c=\frac{8}{3}}$$

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

*Accumulating Function

*derivative of integral in terms of x

Example: $\int_4^x \sqrt{t}\,dt = \int_4^x t^{1/2}\,dt$

$\left[\frac{2}{3}t^{3/2}\right]_4^x$

$\frac{2}{3}x^{3/2} - \frac{2}{3}\cdot 8$

$F(x) = \frac{2}{3}x^{3/2} - \frac{16}{3}$ ← This will calculate the area anywhere from 4 to x.

Take derivative

$F(x)\ \frac{2}{3}x^{3/2} - \frac{16}{3}$

$F'(x) = x^{1/2}$

$F'(x) = \sqrt{x}$

$\therefore \frac{d}{dx}\left[\int_4^x \sqrt{t}\,dt\right] = \sqrt{x}$

Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

pg 329; 96 $\int_1^x \frac{t^2}{t^2+1}\,dt$

$F'(x) = \frac{x^2}{x^2+1}$

pg 329; 100 $\int_0^x \sec^3 t\,dt$

$F'(x) = \sec^3 x$

HOMEWORK

pg 327 - 329; 41, 43, 47, 49, 52, 57, 61, 95, 97, 99